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Research of robust stability of a control system by $m \times n$ linear objects by the method of A.M. Lyapunov functions.

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***Abstract* - On the basis of geometrical interpretation of the second method of Lyapunov theorem a new approach to construction of Lyapunov functions as vector – function which anti-gradient is set by components of a vector of speed (the right member of equation of a state) systems is offered. Stability region of a linear stationary control system is shown as the simplest in equations for own values of a matrix of the closed system with uncertain parameters. Research of robust stability of system is made by designing of the negative function some a sign equal to a scalar product of a vector of gradients on a vector of speed. Condition of stability can be obtained from the positive definiteness of Lyapunov function in the form of system of inequalities in uncertain parameters of control objects and established parameters of the regulator.**

Keywords* - *Control systems, robust stability, superstability, Lyapunov's direct method, modelling, simulation.

I. INTRODUCTION

For modern control problems characteristic of all the increasing complexity of control objects, the requirement of high efficiency and stability in the conditions of numerous uncertainties and incomplete information. Robust stability can be viewed as one of the outstanding issues in control theory, which is also of a great practical interest. Control system design is one of the main tasks in automation of all branches of industry, including machine manufacturing, energy sector, electronics, chemical and biological, metallurgical, textile, transportation, robotics, aviation, space systems, high-precision military systems, etc. In these systems, the uncertainty can be caused by the presence of uncontrolled disturbances acting on an object control [1] and ignorance of the true values of the parameters of control objects and unpredictable change them in time [1,2].

Actual problem is the construction of control systems providing in some sense the best protection against uncertainty in knowledge of the subject. Ability of a control system to keep stability in the conditions of parametrical or nonparametric uncertainty are understood as a system robust.

In the general statement system research on robust stability consists in the instruction restriction on change of parameters of a control system [2]. Considerable number of works is devoted to development of control system robust stability. These works [3,4] mainly study robust stability of polynomial and matrix within the frames of linear development concept of stability of continuous and sampled-data control systems. Universal methods of research of robust stability of dynamic system is absent [4,5,6,7].

In this paper we propose a new approach to the construction of the Lyapunov vector - functions [5,8]. Components of a vector of an anti-gradient, Lyapunov's vector function are set from geometrical interpretation of the theorem of the second method of Lyapunov by [9,10] components of a vector of speed (the right member of equation of a state). Research of robust stability of system is made by designing of the negative function some a sign equal to a scalar product of a vector of gradients on a vector of speed [9,10]. Condition of stability can be obtained from the positive definiteness of Lyapunov function in the form of system of inequalities in uncertain parameters of control objects and established parameters of the regulator.

II MATHEMATICAL MODEL FORMULATION

Let the control system described by the equation of state.

$$\dot{x} = Ax + Bu, x \in R^n, u \in R^m \tag{1}$$

$y \in R^{\ell}$

The regulator is described by the equation

$$u = -kx \tag{2}$$

Here

$$A = \begin{Vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{Vmatrix}, B = \begin{Vmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nm} \end{Vmatrix}.$$

$$C=\begin{Vmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{22} & c_{22} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots \\ c_{l1} & c_{l2} & \dots & c_{\ln} \end{Vmatrix}, x=\begin{Vmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Vmatrix}, y=\begin{Vmatrix} y_1 \\ y_2 \\ \vdots \\ y_l \end{Vmatrix},$$

$$k=\begin{Vmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2n} \\ \dots & \dots & \dots & \dots \\ k_{m1} & k_{m2} & \dots & k_{mn} \end{Vmatrix}, u=\begin{Vmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{Vmatrix},$$

$$u_i=-k_{i1}x_1-k_{i2}x_2-\ldots-k_{in}x_n, i=1,2,\ldots,n$$

A control object matrix can be reduced with the help of no particular matrix P which columns are own functions of A, to the block-diagonal form [11]

$$\widetilde{A}=P^{-1}AP=diag\{\Lambda, \quad _1,\ldots,J_{m.}J_1^{'},\ldots,J_k^{'} \ \} \tag{3}$$

c diagonal square blocks of the form

$$\Lambda=diag\{\lambda_1,\ldots,\lambda_l\}, \tag{4}$$

$$J_j=\begin{Vmatrix} \lambda_j & 1 & \dots & 0 & 0 \\ 0 & \lambda_j & \dots & 0 & 0 \\ \dots & . & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_j & 1 \\ 0 & 0 & \dots & 0 & \lambda_j \end{Vmatrix}, N_j\times N_j\, j=1,\ldots,m, \tag{5}$$

$$J_j^{'}=\begin{Vmatrix} \alpha_j & -\beta_j \\ \beta_j & \alpha_j \end{Vmatrix}, j=1,\ldots,k. \tag{6}$$

where $\lambda_1,\ldots,\lambda_l$ - real simple, $\lambda_j -$ real, N_j multiple, $\lambda_j=\alpha_j\pm j\beta_j -$ complex own values of a matrix A, and it is obvious $l+N_1+\ldots+N_m+2k=n.$

We show that the accepted structure (3) allows separate control research by initial representations of the object (4), (5) and (6) corresponding to any block diagonal matrix $\widetilde{A}$ For this purpose like (1) we will write down.

$$\widetilde{x}=\widetilde{A}\widetilde{x}+\widetilde{B}u=\begin{Vmatrix} \Lambda & & 0 \\ & J & \\ 0 & & J^{'} \end{Vmatrix}\widetilde{x}+\begin{Vmatrix} \widetilde{B}_1 \\ \widetilde{B}_2 \\ \widetilde{B}_3 \end{Vmatrix}\widetilde{u} \tag{7}$$

$$u=-\widetilde{k}\widetilde{x} \tag{8}$$

where

$$\widetilde{x}=P^{-1}x, \quad \widetilde{A}=P^{-1}AP \ \widetilde{B}=p^{-1}B, \widetilde{k}=kP$$

and thus dimensions of matrixes $\widetilde{B}_1,\widetilde{B}_2 u \ \widetilde{B}_3$ and a vector of control u correspond to dimensions of square matrixes $\Lambda, J, J^{'}$. On the basis of (7) having accepted $\widetilde{B}_2=0,\ \widetilde{B}_3=0$ it is easy to be convinced, that we can operate the coordinates of system (7) corresponding to a matrix Λ, keeping invariable coordinates of system (7) defined by matrixes $J_u J^{'}$, respectively $\widetilde{B}_1=0 \ \ u \ \ \widetilde{B}_3=0$ or $\widetilde{B}_1=0$ и $\widetilde{B}_2=0$. Thus, the further task is reduced to consecutive research of robust stability of linear control systems for initial objects

$$\dot{x}_1=\Lambda\widetilde{x}_1+\widetilde{B}_1u \tag{9}$$

$$\dot{x}_2=J\widetilde{x}_2+\widetilde{B}_2u \tag{10}$$

$$\dot{x}_3=J\widetilde{x}_3+\widetilde{B}_3u \tag{11}$$

where

$$\widetilde{x}_1=\begin{Vmatrix} \widetilde{x}_1 \\ \widetilde{x}_2 \\ \vdots \\ x_l \end{Vmatrix}, \widetilde{x}_2=\begin{Vmatrix} \widetilde{x}_{l+1} \\ \widetilde{x}_{l+2} \\ \vdots \\ x_{l+L} \end{Vmatrix}, L=N_1+\ldots+N_{m,}\widetilde{x}_3=\begin{Vmatrix} \widetilde{x}_{l+L+1} \\ x_{l+L+2} \\ \vdots \\ \widetilde{x}_n \end{Vmatrix},$$

With matrixes (4) – (6). We will consider serially a research problem of robust stability (9) - (10) method of Lyapunov functions. We will assume, for simplicity of record and presentation that

$$B=\begin{Vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{Vmatrix}, u\in R,\ u=-k^T x,\quad k=\begin{Vmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{Vmatrix}$$

Like (7) we will write down

$$\widetilde{x}=\widetilde{A}\widetilde{x}\quad+\widetilde{b}\,u=\begin{Vmatrix} \Lambda & & 0 \\ & J & \\ 0 & & J^{'} \end{Vmatrix}\widetilde{x}+\begin{Vmatrix} \widetilde{b}_1 \\ \widetilde{b}_2 \\ \widetilde{b}_3 \end{Vmatrix}\widetilde{u}, \tag{12}$$

$$\widetilde{u}=-\widetilde{k}^T\widetilde{x}=-\begin{Vmatrix}\widetilde{k}_1^T\widetilde{k}_2^T\widetilde{k}_3^T\end{Vmatrix}\widetilde{x}, \tag{13}$$

where

$$\widetilde{x}=P^{-1}x, \widetilde{A}=P^{-1}A\,P,\quad b=P^{-1}b,\quad \widetilde{k}^T=k^T P,$$

and thus dimensions of matrixes columns $\widetilde{b}_1,\widetilde{b}_2,\widetilde{b}_3$ and matrixes lines $\widetilde{k}_1^T,\widetilde{k}_2^T,\widetilde{k}_3^T$ correspond to dimensions of square matrixes $\Lambda, J, J^{'}$. На основании (12), (13), (14), having accepted $\widetilde{k}_2^T=0,\ \ \widetilde{k}_3^T=0,$ it isn't difficult to receive characteristic determinant of the closed system

$$\left|\lambda I-(\widetilde{A}-\widetilde{b}\widetilde{k}^T)\right|=$$

$$\left|\lambda I_1-(\Lambda-\widetilde{b}_1\widetilde{k}_1^T)\right|.\left|\lambda I_2-J\right| \ . \ \left|\lambda I_3-J^{'}\ \right|$$

from which it is obvious that changing coefficients of a matrix of the regulator $\widetilde{k}_1^T$ it is possible to operate own values of a matrix $D_1=(\Lambda-\widetilde{B}_1\widetilde{k}_1^T)$, keeping invariable own values of a matrix J or $J^{'}$, respectively.

Having accepted $\widetilde{k}_1^T=0,\ \ \widetilde{k}_3^T=0$ or $\widetilde{k}_1^T=0,\quad \widetilde{k}_2^T=0.$ Thus, it allows consecutive consideration of initial objects similarly (9), (10), (11).

$$\dot{\widetilde{x}}=\Lambda\widetilde{x}+\widetilde{b}_1u \tag{14}$$

$$\dot{\tilde{x}} = \Lambda\tilde{x} + \tilde{b}_2 u \qquad (15)$$

$$\dot{\tilde{x}} = \Lambda\tilde{x} + \tilde{b}_3 u \qquad (16)$$

III STABILITY CONDITIONS OF THE STEADY STATES OF THE SYSTEM

A. CASE OF REAL ROOTS

Thus we will assume own numbers of A_1 matrix are set and equal $\tilde{A}_1 = diag\{s_1, s_2, ..., s_n\}$. Such basis is convenient to that in it the equations of system (14) break up to the equations of independent subsystems of the first order concerning components of a vector x. We will receive

$$\begin{cases} \dot{\tilde{x}}_1 = (s_1 - \tilde{b}_1\tilde{k}_1)\tilde{x}_1 \\ \dot{\tilde{x}}_2 = (s_2 - \tilde{b}_2\tilde{k}_2)\tilde{x}_2 \\ \vdots \\ \dot{\tilde{x}}_l = (s_l - \tilde{b}_l\tilde{k}_l)\tilde{x}_l \end{cases} \qquad (17)$$

It is known at researches of stability of system of the equation of a state registers in deviations Δx relatively some steady state $X_s\,(x = \Delta x = X(t) - X_s(t))$. Thus, a speed vector (defined by the state equation) it is directed in steady system always by the beginning of coordinates (zero). From geometrical interpretations of A.M. Lyapunov's theorems we can assume that gradient's vectors are sent to the opposite side of the greatest growth required Lyapunov's functions, but they are equal in size. Lyapunov's functions we are set in the form of scalar functions $V(x_1, x_2, ..., x_l)$, and from Lyapunov's functions we can present a gradient:

$$\frac{\partial V(\tilde{x})}{\partial \tilde{x}} = -\frac{d\tilde{x}_1}{dt} = -(A_1 - \tilde{b}_1\tilde{k}_1^T)\tilde{x}_1$$

Then components of a vector of a gradient from potential functions $V(x_1, x_2, ..., x_l)$, we can present in a following view:

$$-\frac{d\tilde{x}_1}{dt} = -\frac{\partial V(\tilde{x})}{\partial \tilde{x}_1}, -\frac{d\tilde{x}_2}{dt} = \frac{\partial V(\tilde{x})}{\partial \tilde{x}_2}, ..., -\frac{d\tilde{x}_l}{dt} = \frac{\partial V(\tilde{x})}{\partial x_l}, \qquad (18)$$

Full derivative on time from Lyapunov's functions according to the theorem of asymptotic stability taking into account the equations the state (17), in steady system has to be negative function and is defined as a scalar product of a gradient's vector from Lyapunov's functions on a vector of speed of system i.e.

$$\begin{aligned} \frac{dV(\tilde{x})}{dt} &= -\sum_{i=1}^{l} \frac{\partial V(\tilde{x})}{\partial \tilde{x}_i}\frac{d\tilde{x}_i}{dt} = -[(s_1 - \tilde{b}_1\tilde{k}_1)\tilde{x}_1]^2 - \\ &-[(s_2 - \tilde{b}_2\tilde{k}_2)\tilde{x}_2]^2 - \ldots - [(s_l - \tilde{b}_l\tilde{k}_l)\tilde{x}_l]^2 = \\ &= -\sum_{i=1}^{l} (s_i - \tilde{b}_i\tilde{k}_i)^2\,\tilde{x}_i^2 \end{aligned} \qquad (19)$$

From (19) it is visible that full derivative of Lyapunov's functions on time will be always negative function, i.e. it will be satisfied conditions of asymptotic stability of system at positive distinctness of Lyapunov's functions.

From (18) we will receive that

$$\frac{\partial V(\tilde{x})}{\partial \tilde{x}_1} = -(s_1 - \tilde{b}_1\tilde{k}_1)\tilde{x}_1,$$

$$\frac{\partial V(\tilde{x})}{\partial \tilde{x}_2} = -(s_2 - \tilde{b}_2\tilde{k}_2)\tilde{x}_2, \ldots, \ldots \frac{\partial V(\tilde{x})}{\partial \tilde{x}_l} = -(s_l - \tilde{b}_l\tilde{k}_l)\tilde{x}_l,$$

From here we can receive Lyapunov's functions in a following view:

$$\begin{aligned} V(\tilde{x}) = &-(s_1 - \tilde{b}_1\tilde{k}_1)\tilde{x}_1^2 - (s_2 - \tilde{b}_2\tilde{k}_2)\tilde{x}_2^2 - \\ &- \ldots - (s_l - \tilde{b}_l\tilde{k}_l)\tilde{x}_l^2 \end{aligned} \qquad (20)$$

Conditions of positive distinctness of Lyapunov's functions has an appearance:

$$\begin{cases} (s_1 - \tilde{b}_1\tilde{k}_1) < 0 \\ (s_2 - \tilde{b}_2\tilde{k}_2) < 0 \\ \quad\ldots \\ (s_l - \tilde{b}_l\tilde{k}_l) < 0 \end{cases} \qquad (21)$$

In system of inequalities (21) expressions $\mu_i = s_i - \tilde{b}_i\tilde{k}_i,\ i = 1, \ldots, l$ is own values of a matrix of the closed system, and received known result of the linear principle of stability $\mu_i = s_i - \tilde{b}_i\tilde{k}_i < 0,\ i = 1, \ldots, l.$

B. CASE OF MULTIPLE ROOTS

Let the matrix A about L have multiple own numbers: s_1 - multiplicity N_1, s_2 - multiplicity $N_2, \ldots,$ s_m - multiplicity N_m. The condition is satisfied $\sum_{i=1}^{m} N_i = L$ Zhordan's cells for own numbers $(\mathrm{I}_m s_j = 0,\ j = 1, \ldots, m)$ has an appearance (5). (15) equation we will construct for Zhordano's one block in the developed form:

$$\begin{cases} \dot{\tilde{x}}_i = s_j\tilde{x}_i + \tilde{x}_{i+1} - \tilde{b}_i\tilde{k}_i\tilde{x}_i \\ \dot{\tilde{x}}_{i+1} = s_j\tilde{x}_{i+1} + \tilde{x}_{i+2} - \tilde{b}_{i+1}\tilde{k}_{i+1}\tilde{x}_{i+1}. \\ \quad\ldots \\ \dot{\tilde{x}}_{i+N_i} = s_j\tilde{x}_{i+N_i} - \tilde{b}_{i+N_i}\tilde{k}_{i+N_i}\tilde{x}_{i+N_i} \end{cases} \qquad (22)$$

$j = 1, 2, \ldots, m,\ i = l+1, \ldots l+L$

Lyapunov's functions we build in a view a vector of functions with components $V_i(\tilde{x}), V_{i+1}(\tilde{x}), \ldots, \ldots V_{i+N_i}(\tilde{x})$, and gradients from components of Lyapunov's functions it is representable in a following view

$$\begin{cases} -\dfrac{d\tilde{x}_i}{dt} = \dfrac{\partial V_i(\tilde{x})}{\partial \tilde{x}_i} + \dfrac{\partial V_i(\tilde{x})}{\partial \tilde{x}_{i+1}} \\ -\dfrac{d\tilde{x}_{i+1}}{dt} = \dfrac{\partial V_{i+1}(\tilde{x})}{\partial \tilde{x}_{i+1}} + \dfrac{\partial V_{i+1}(\tilde{x})}{\partial \tilde{x}_{i+2}} \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ -\dfrac{d\tilde{x}_{i+N_i}}{dt} = \dfrac{\partial V_{i+N_i}(\tilde{x})}{\partial \tilde{x}_{i+N_i}} \end{cases}$$

This system we can, substituting value from (22) to copy in a following view

$$-s_j\tilde{x}_i-\tilde{x}_{i+1}+\tilde{b}_i\tilde{k}_i\tilde{x}_i=\frac{\partial V_i(\tilde{x})}{\partial\tilde{x}_i}+\frac{\partial V_i(\tilde{x})}{\partial x_{i+1}}$$

$$-s_j\tilde{x}_{i+1}-\tilde{x}_{i+2}+\tilde{b}_{i+1}\tilde{k}_{i+1}\tilde{x}_{i+1}=\frac{\partial V_{i+1}(\tilde{x})}{\partial\tilde{x}_{i+1}}+\frac{\partial V_{i+1}(\tilde{x})}{\partial\tilde{x}_{i+2}} \quad (23)$$

... ...

$$-s_j\tilde{x}_{i+n_i}+b_{i+N_i}\tilde{k}_{i+N_i}\tilde{x}_{i+N_i}=\frac{\partial V_{i+N_i}(\tilde{x})}{\partial\tilde{x}_{i+N_i}}$$

From here from (23) we will receive full derivatives on time from components of Lyapunov's vector functions

$$\frac{dV_i(\tilde{x})}{dt}=-(s_j\tilde{x}_i+\tilde{x}_{i+1}-\tilde{b}_i\tilde{k}_i\tilde{x}_i)^2$$

$$\frac{dV_{i+1}(\tilde{x})}{dt}=-(s_j\tilde{x}_{i+1}+\tilde{x}_{i+2}-\tilde{b}_{i+1}\tilde{k}_{i+1}\tilde{x}_{i+1})^2$$

... ...

$$\frac{dV_{i+N_i}(\tilde{x})}{dt}=-(s_j\tilde{x}_{i+N_i}-b_{i+N_i}\tilde{k}_{i+N_i}\tilde{x}_{i+N_i})^2$$

i.e. full derivatives on time will be negative function (24) and satisfies conditions asymptotic stability. Also from (23) we can find values of components of a vector of a gradient:

$$\begin{cases}\frac{\partial V_i(\tilde{x})}{\partial\tilde{x}_i}=-(s_j-\tilde{b}_i\tilde{k}_i)\tilde{x}_i;\frac{\partial V_{i_1}(\tilde{x})}{\partial\tilde{x}_{i+1}}=-\tilde{x}_{i+1}\\ \frac{\partial V_{i+1}(\tilde{x})}{\partial\tilde{x}_{i+1}}=-(s_j-\tilde{b}_{i+1}\tilde{k}_{i+1})\tilde{x}_{i+1},\frac{\partial V_{i+1}}{\partial x_{i+2}}=-\tilde{x}_{i+2}\\ \dots\quad\dots\quad\dots\quad\dots\quad\dots\\ \frac{\partial V_{i+N_i}(\tilde{x})}{\partial\tilde{x}_{i+N_i}}=-(s_j-b_{i+N_i}k_{i+N_i})\tilde{x}_{i+N_i}\end{cases} \quad (24)$$

From here we can receive components of Lyapunov's vector functions

$$V_i(\tilde{x})=-(s_j-\tilde{b}_i\tilde{k}_i)\tilde{x}_i^2-\tilde{x}_{i+1}^2$$

$$V_{i+1}(\tilde{x})=-(s_j-\tilde{b}_{i+1}\tilde{k}_{i+1})\tilde{x}_{i+1}^2-x_{i+2}^2$$

... .

$$V_{i+N_i-1}(\tilde{x})=-(s_j-b_{i+N_{i-1}}\tilde{k}_{i+N_i}-1)\tilde{x}_{i+N_{i-1}}^2-x_{i+N_i}^2$$

$$V_{i+N_i}(\tilde{x})=-(s_j-\tilde{b}_{i+N}\tilde{k}_{i+N_i})\tilde{x}_{i+N_i}^2$$

We can present Lyapunov's function in the form of scalar functions

$$V(\tilde{x})=-(s_j-\tilde{b}_i\tilde{k}_i)\tilde{x}_i^2-(s_j+1-\tilde{b}_{i+1}\tilde{k}_{i+1})\tilde{x}_{i+1}^2$$
$$-(s_j+1-\tilde{b}_{i+2}\tilde{k}_{i+2})\tilde{x}_{i+2}^2-\ldots-(s_j+1-\tilde{b}_{i+N_i}\tilde{k}_{i+N_i})\tilde{x}_{i+N_i}^2 \quad (25)$$

From here the condition of positive distinctness of functions of Lyapunov (25) will be expressed by system of inequalities

$$\begin{cases}-(s_j-\tilde{b}_i\tilde{k}_i)>0\\ -(s_j+1-\tilde{b}_{i+1}\tilde{k}_{i+1})>0\\ -(s_j+1-\tilde{b}_{i+2}\tilde{k}_{i+2})>0\\ \dots\quad\dots\quad\dots\quad\dots\\ -(s_j+1-\tilde{b}_{i+N_i}\tilde{k}_{i+N_i})>0\end{cases}$$

Or we can rewrite in a following view

$$\begin{cases}s_j-\tilde{b}_i\tilde{k}<0\\ s_j+1-\tilde{b}_{i+1}\tilde{k}_{i+1}<0\\ s_j+1-\tilde{b}_{i+2}\tilde{k}_{i+2}<0\\ \dots\quad\dots\quad\dots\quad\dots\\ s_j+1-\tilde{b}_{i+N_i}\tilde{k}_{i+N_i}<0\end{cases} \quad (26)$$

The system of inequalities (26) also expresses a condition of negativity of own values of a matrix of the closed system.

C. CASE OF COMPLEX ROOTS

Let the matrix of control objects A of dimension 2k have complex own values

$$\lambda_i=\alpha_i\pm j\beta_i,\ i=1,\ldots,k$$

(16) equation we will construct for one block in the developed form:

$$\begin{cases}\tilde{x}_i=\alpha_i\tilde{x}_i+\beta_i x_{i+1}-\tilde{b}_i\tilde{k}_i\tilde{x}_i\\ \tilde{x}_{i+1}=-\beta_i x_i+\alpha_i\tilde{x}_{i+1}-\tilde{b}_{i+1}\tilde{k}_{i+1}\tilde{x}_{i+1}\end{cases}\quad i=1,\ldots,k \quad (27)$$

Lyapunov's functions we construct in the form of vector functions with $V_i(\tilde{x})\ u\ V_{i+1}(\tilde{x})$, components and gradient components from components the vector of Lyapunov's functions is representable

$$\begin{cases}-\frac{d\tilde{x}_i}{dt}=\frac{\partial v_i(\tilde{x})}{\partial\tilde{x}_i}+\frac{\partial v_i(\tilde{x})}{\partial\tilde{x}_{i+1}}\\ -\frac{d\tilde{x}_{i+1}}{dt}=\frac{\partial v_{i+1}(\tilde{x})}{\partial\tilde{x}_i}+\frac{\partial v_{i+1}(\tilde{x})}{\partial\tilde{x}_{i+1}}\end{cases}\quad i=1,\ldots,k \quad (28)$$

Substituting from (27) value system (28) we will copy in a following view

$$\begin{cases}-\alpha_i\tilde{x}_i+\beta_i x_{i+1}-\tilde{b}_i\tilde{k}_i\tilde{x}_i=\frac{\partial v_i(\tilde{x})}{\partial\tilde{x}_i}+\frac{\partial v_i(\tilde{x})}{\partial\tilde{x}_{i+1}}\\ +\beta_i\tilde{x}_i+\alpha_i\tilde{x}_{i+1}-\tilde{b}_{i+1}\tilde{k}_{i+1}\tilde{x}=\frac{\partial v_{i+1}(\tilde{x})}{\partial\tilde{x}_i}+\frac{\partial v_{i+1}(\tilde{x})}{\partial\tilde{x}_{i+1}}\end{cases} \quad (29)$$

From (29) we will receive full derivatives on time from components of Lyapunov's vector functions

$$\begin{cases}\frac{dv_i(\tilde{x})}{dt}=-(\alpha_i\tilde{x}_i+\tilde{\beta}_i\tilde{x}_{i+1}-\tilde{b}_i\tilde{k}_i\tilde{x}_i)^2\\ \frac{dv_{i+1}(\tilde{x})}{dt}=-(-\beta_i x_i+\alpha_i x_{i+1}-\tilde{b}_{i+1}\tilde{k}_{i+1}\tilde{x}_{i+1})^2\end{cases}$$
$$i=1,\ldots,k \quad (30)$$

Full derivatives on time from a component Lyapunov's vector - functions are always negative function and conditions of asymptotic stability is carried out.

From (29) we can find values of components a vector - a gradient:

$$\frac{\partial V_i(\tilde{x})}{\partial \tilde{x}_i} = -(\alpha_i - \tilde{b}_i\tilde{k}_i\tilde{x}_i)\tilde{x}_i; \; \frac{\partial V_i(\tilde{x})}{\partial \tilde{x}_{i+1}} = \beta_i\tilde{x}_{i+1}$$

$$\frac{\partial V_{i+1}(\tilde{x})}{\partial \tilde{x}_i} = \beta_i\tilde{x}_i; \; \frac{\partial V_{i+1}(\tilde{x})}{\partial \tilde{x}_{i+1}} = -(\alpha_i - \tilde{b}_{i+1}\tilde{k}_{i+1})\tilde{x}_{i+1};$$

We will receive components a Lyapunov's vector - functions in a following view

$$V_i(\tilde{x}) = -(\alpha_i - \tilde{b}_i\tilde{k}_i)\tilde{x}_i^{\,2} - \beta_i\tilde{x}_{i+1}^{\,2}$$

$$V_{i+1}(\tilde{x}) = -(\alpha_i - \tilde{b}_{i+1}\tilde{k}_{i+1})\tilde{x}_{i+1}^{\,2} + \beta_i\tilde{x}_i^{\,2}$$

In a scalar form we can present Lyapunov's function in a following view

$$V(\tilde{x}) = -(\alpha_i - \tilde{b}_i\tilde{k}_i)\tilde{x}_i^2 - (\alpha_i - \tilde{b}_{i+1}\tilde{k}_{i+1})\tilde{x}_{i+1}^2 + + \beta_i(\tilde{x}_i^2 - \tilde{x}_{i+1}^2)$$

Here we can accept $\tilde{x}_i = \tilde{x}_{i+1}; \tilde{b}_i k_i = \tilde{b}_{i+1}\tilde{k}_{i+1,}$ then we will receive $V(\tilde{x}) = -2(\alpha_i - \tilde{b}_i\tilde{k}_i)\tilde{x}_i^2, i = 1,...,n$

Conditions of positive distinctness of Lyapunov's functions will be it is expressed by system of inequalities

$$\alpha_i - \tilde{b}_i\tilde{k}_i < 0, \quad i = 1,...,k \qquad (31)$$

Conditions (31) also are expressed by negativity of the valid part of own values $\mu_i = \alpha_i - \tilde{b}_i\tilde{k}_i < 0,\ i = 1,..., k$ matrixes of the closed system. If necessary it is possible to determine the radius of robust stability.

IV CONCLUSION

In this paper robust stability perform an important function in the theory of control of dynamic objects and described in [5,13-16].

Results showed that for research of robust stability of control systems the new method of the creation of A.M Lyapunov function on the basis of geometrical interpretation of Lyapunov's function can be applied with a great success. A.M Lyapunov function is synthesized in the form of the vector function anti-gradient of which is set in the form of a speed vector. Area of stability of stationary conditions of system are received in the form of the elementary inequalities.

This method is an extension of the notion of stability where the Lyapunov function is replaced by a geometric interpretation of the Lyapunov function with dependence on the uncertain parameters [16-18].

The radius of stability coefficients interval family of positive definite functions is equal to the smallest value of the coefficients of the vector Lyapunov functions. Thus, a condition of stability of the linear closed system is the negative sign of material part of all proper values of a matrix of the closed system. The received results confirm the linear principle of stability of a system.

Theoretical results obtained in this paper are an important contribution to the theory of stability, to the theory of robust stability of linear control systems.

Thus, for a wide class of systems, we believe the theory is sufficiently well developed that work can begin on developing efficient approach to aid control engineers in incorporating the parametric approach into their analysis and design toolboxes.

The practical importance of these results should motivate new theoretical studies on typical application techniques, clarification area of the robust control and design complex automated system [18-20].

Finally, this is the main results that theoretical approaches represent the most promising direction. These studies are especially important for the designing more effective automation control systems.

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